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Rigid Body Motion: The End Effector of 4-DoF Robot Motion Analysed by Denavit Hartenberg Method

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ABSTRACT

Rigid body motion is consist of two kinds motion which are rotational and translational motion, as an example is a robot kinematic motion. Robot forward kinematics refers to its final position (end effector), which combines rotational and translational motion along its trajectory. This research is a theoretical physics study that aims to analyze the end effector of a robot arm with four degrees of freedom (4-DoF) using the Denavit Hartenberg (DH) method and visualize the motion by using Simulink (DH-parameters) in MATLAB. This research was carried out through manual analysis and visualization of the equations obtained. This research provides the transformation matrix g_st (θ) as a forward kinematics equation and visualize the motion by MATLAB.

INTRODUCTION

One type of robot widely used on an industrial scale is the robotic manipulator (Faiq, et al., 2022; Suddin, et al., 2017) The development of robot manipulators is expected to make work easier and more efficient. In general, a robot manipulator is composed of an arm (link), a joint, and a hand (as an end-effector) in the form of a hand with a finger (gripper) and degrees of freedom (DoF) as a combination of movements from each joint. (joints). Many studies have been carried out related to the development of robotic manipulator systems (Faiq, et al., 2022; Suddin, et al., 2017; Ge, 2022; Gallardo-Alvarado, et al., 2018; Purwanto, et al., 2020; Salman and Roman, 2022; Kucuk and Bingul, 2006; Serrezuela, et al., 2017)

Research conducted by Utomo and Munadi (2013) stated that the motion of robot manipulators is carried out by analyzing robot kinematics. Research from Ge (2022) analyzes robot kinematics using screw theory, Newton Raphson's method, and DH parameters in MATLAB, showing that this method has high accuracy and is more practical for analyzing forward kinematics on a robot arm (manipulator) with 7 degrees of freedom. The results of a study by Andika and Salamah (2018) state that the analysis of forward and reverse kinematics on a robotic arm (manipulator) with 3 degrees of freedom can be completed using the DH-parameters. This research was obtained using MATLAB and RoboAnalyzer software, which shows that the DH-parameters can formulate forward kinematics and reverse kinematics on a robot arm (manipulator) with 3 degrees of freedom and produce the same matrix from manual calculations or via MATLAB and RoboAnalyzer software.

Generally, robot motion can be analyzed based on the concept of motion in Physics, i.e., the concept of motion of a rigid body. The system's motion is said to be a rigid body motion if the distance between points on each body does not change during the motion. This concept is becoming the basic concept to analyze the motion of robotic manipulators

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(Murray, et al., 1994; Yasmini and Gunadi, 2012). This research analyses the basic concepts of kinematics in robot arm movement using DH-parameters and visualizes by using MATLAB.

RESEARCH METHOD

This research is a theoretical physics study carried out through a literature review of previously developed procedures, analyses manually to obtain an equation. Next, to compare the results and visualize the equation, thus providing an overview of the form of the equation that has been studied. The research method used is a literature study and manual analysis, which are then visualized using MATLAB.

RESULTS AND DISCUSSION

Robot Manipulator 4-DoF and Denavit Hartenberg Related Parameters

In this study, the author analyses the basic concepts of robot kinematics in moving a robot arm (manipulator) with 4-DoF using the DH parameters represented in Figure 1.

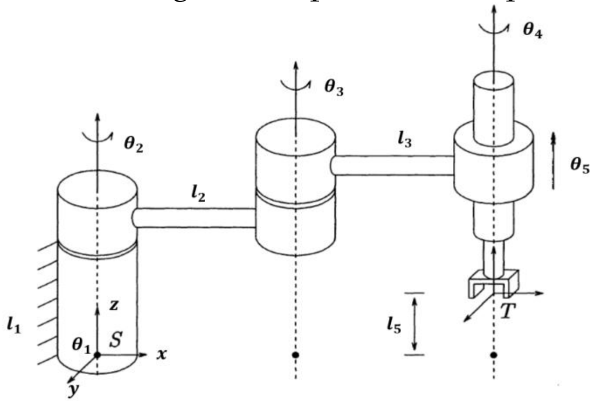

Figure 1. Robot manipulator with 4-DoF [10]

The representation of a robot manipulator with 4-DoF in Figure 1 shows that the system configuration consists of a pair (p_{st}, R_{st}) where p_{st} shows the movement journey until the end effector point of the robot is found or can be called a position vector in the T frame which originates from the frame S. So that the motion of a rigid body can be represented using a rigid body transformation to describe the instantaneous position and orientation of the body's coordinate frame relative to the fixed frame. A rotation matrix that represents the movement configuration of point S towards point T is represented by $g_{st} \in SO(3)$, so the mapping g_{st} can be expressed as follows.

$$g_{st}(\theta) = g_{l_0 l_1}(\theta_1) g_{l_1 l_2}(\theta_2) \cdots g_{l_{n-1,} l_n}(\theta_n) g_{l_n t}, = \begin{bmatrix} R(\theta) & p(\theta) \\ 0 & 1 \end{bmatrix} \quad (1)$$

Equation (1) shows the relationship between the geometric concept of spatial joints in robots and the general coordinate concept used to determine the position of an object. The goal of kinematics is to define the relative position of a frame with its original coordinates. Based on this, Figure 2 represents the coordinate configuration of each frame on a robot manipulator with 4-DoF as follows,

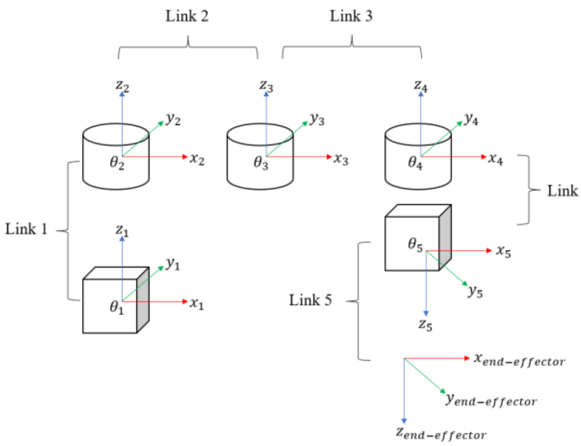

Figure 2. Configure the coordinates of each frame on the robot manipulator with 4-DoF

Figure 2 shows the angular position and direction of each joint and link movement, which can be determined using the concept of a rigid body. The motion of a rigid body consists of rotational motion about a straight line followed by translational motion on that straight line using exponential coordinates. Figure 2 also provides a representation of the primary frame (S or θ_1) and final frame or end effector (T) with the corresponding rotation coordinates at each joint of the robot, thus providing a complete parameterization of the kinematics of the robot.

An alternative parameterization, a standard commonly used in robotics, is the Denavit Hartenberg parameters [10]. The Denavit Hartenberg parameters in each homogeneous transformation g_{l_{i-1},l_i} according to equation (1), is represented by the product of four basic transformations, which can be seen in the following equations,

$$g_{l_{i-1},l_i} = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i} \tag{2}$$

$$g_{l_{i-1},l_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i & 0 \\ 0 & -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$g_{l_{i-1},l_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Based on equation (4), it is known that there are four parameters in Denavit Hartenberg, namely θ, α, d, and a are parameters from *the link* i and *joint* to i, which have been represented in Figure 2 regarding the coordinate configuration of each frame on the robot manipulator with 4-DoF. After determining the coordinates of each frame, determine the DH-parameters for the robot, shown in Table 1.

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Table 1. DH-parameters for robot manipulator with 4-DoF motion

Link	θ	d	a	α	Information
1	0	l_1	0	0	Base
2	θ_2	0	l_2	0	Arm
3	θ_3	0	l_3	0	Arm
4	θ_4	0	0	180°	Arm
5	0	l_5	0	0	end-effector

3.2 Kinematic Analysis

The forward kinematics equation of motion of the robot arm under study can be formulated with the values given based on Table 1 regarding the DH-parameters of the coordinate configuration of each frame on the robot manipulator with 4-DoF. So, the homogeneous matrix transformation for each arm can be determined using equation (4) as follows. Matrix transformation for link 1 (g_{l_1,l_2}).

$$g_{l_1,l_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

Matrix transformation for link 2 (g_{l_2,l_3}).

$$g_{l_2,l_3} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

Matrix transformation for link 3 (g_{l_3,l_4}).

$$g_{l_3,l_4} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & l_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & l_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Matrix transformation for link 4 (g_{l_4,l_5}).

$$g_{l_4,l_5} = \begin{bmatrix} \cos\theta_4 & \sin\theta_4 & 0 & 0 \\ \sin\theta_4 & -\cos\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Matrix transformation for link 5 (g_{l_5,l_1}).

$$g_{l_5,l_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The robot's forward kinematics can be described by referring to the position of the end effector, which is the result of rotational and translational movements along $p(\theta)$ or $p(x)$, $p(y)$ and $p(z)$ (position vectors in the transformation matrix) according to equation (1). So, the homogeneous transformation matrix produced for joint movement from the base to the end effector can be written as follows,

$$g_{st}(\theta) = g_{l_1 l_2}(\theta_2) g_{l_2 l_3}(\theta_3) g_{l_3 l_4}(\theta_4) g_{l_4 l_5}(\theta_5) g_{l_5 l_1}(\theta_1)$$

$$g_{st}(\theta) = \begin{bmatrix} \cos(\theta_2+\theta_3+\theta_4) & \sin(\theta_2+\theta_3+\theta_4) & 0 & l_3 \cos\ (\theta_2+\theta_3) + l_2 \cos\theta_2 \\ \sin(\theta_2+\theta_3+\theta_4) & -\cos(\theta_2+\theta_3+\theta_4) & 0 & l_3 \sin\ (\theta_2+\theta_3) + l_2 \sin\theta_2 \\ 0 & 0 & -1 & -l_5 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

The result of the transformation matrix $g_{st}(\theta)$ as equation (10) is a forward kinematics equation obtained using DH-parameters representing the kinematics of the rigid body, which is studied about the position and orientation of the frame attached to each link of the robot and then building a homogeneous transformation between the frames of the robot manipulator 4-DoF.

3.3 Visualization by MATAB

For visualizing the forward kinematics of the robot manipulator by MATLAB, the author entered Table 1 regarding the DH-parameters for the movement of the robot into the Robotic Toolbox by Peter Corke on MATLAB. As in equation (10), the rotation matrix with the associated angles used is a sum. Whereas the translation matrix is related to the arm's length and the rotation angles of the robot arm movement.

Based on that, it is necessary to create an initial position of the robot before it transforms. The initial state, chosen that l_1, l_2, l_3, and l_5 at 10 cm while θ_2, θ_3, and θ_4 at 0°. Then a state is obtained by determining the value of the DH-parameter in the transformation or a combination of movements of the robot (as in Figure 1 and Figure 2) to set the initial state as follows.

Table 2. DH-parameters describing the robot's initial position.

Link	$\boldsymbol{\theta}$	$\boldsymbol{d}$	$\boldsymbol{a}$	$\boldsymbol{\alpha}$	Information
1	0°	10 cm	0	0	Base
2	0°	0	10 cm	0	Arm
3	0°	0	10 cm	0	Arm
4	0°	0	0	180°	Arm
5	0°	10 cm	0	0	*end-effector*

Based on Table 2, regarding the DH-parameters for the transformation of the robot being studied, the homogeneous matrix transformation for each link or arm can be described using (10),

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$$g_{st}(\theta) = \begin{bmatrix} \cos(\theta_2+\theta_3+\theta_4) & \sin(\theta_2+\theta_3+\theta_4) & 0 & l_3\cos(\theta_2+\theta_3)+l_2\cos\theta_2 \\ \sin(\theta_2+\theta_3+\theta_4) & -\cos(\theta_2+\theta_3+\theta_4) & 0 & l_3\sin(\theta_2+\theta_3)+l_2\sin\theta_2 \\ 0 & 0 & -1 & -l_5+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{st}(\theta) = \begin{bmatrix} cos(0°+0°+0°) & sin(0°+0°+0°) & 0 & 10\,cos(0°+0°)+10\,cos\,0° \\ sin(0°+0°+0°) & -cos(0°+0°+0°) & 0 & 10\,sin(0°+0°)+10\,sin\,0° \\ 0 & 0 & -1 & -10+10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{st}(\theta) = \begin{bmatrix} \cos(0°) & \sin(0°) & 0 & 10\cos(0°)+10\cos 0° \\ \sin(0°) & -\cos(0°) & 0 & 10\sin(0°)+10\sin 0° \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{st}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 10+10 \\ 0 & -1 & 0 & 0-0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{st}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Visualization of the initial position of the robot as shown in Figure 3.

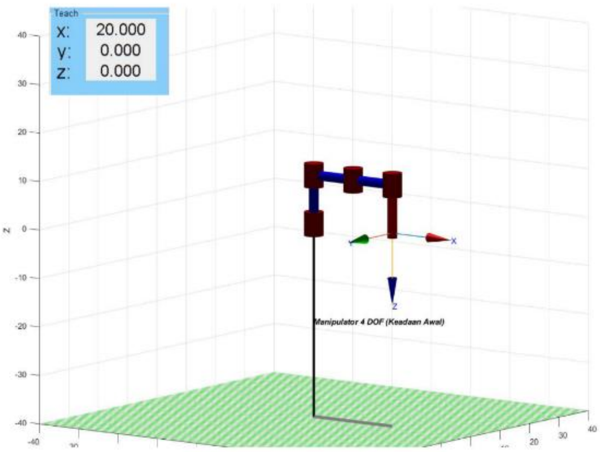

Figure 3. Visualization of the robot's initial position

Figure 3 represents the results of the $g_{st}(\theta)$ transformation equation, that formed from the robot's initial position, that analyzed and visualized by MATLAB. Moreover, we can see that the x, y, and z-axes represent the output result as the end effector position of the robot, form the box we know that $[20 \quad 0 \quad 0]$, respectively. So, this can be used as a comparison or validation of the correctness of the manual analyses that we have carried out.

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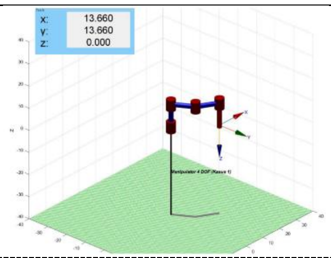
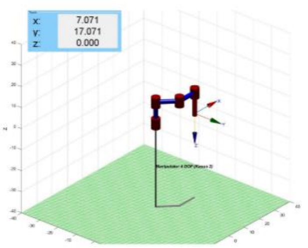
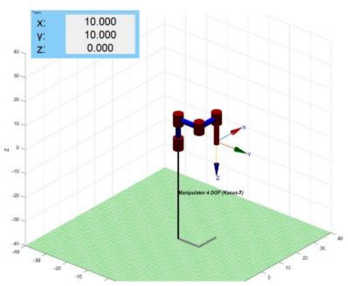
3.4 Case

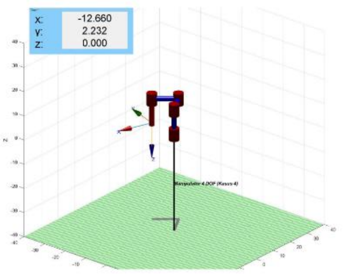
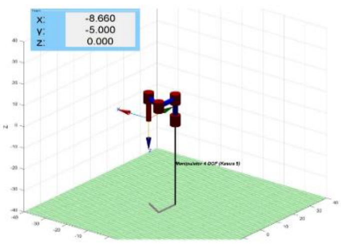
As in Figure 3, each robot's arm or join have to provide the degree of freedom to move regarding the given rotation angle. Based on this, the following is an example of a robotic arm case determined by varying the rotation angle of each joint.

Table 3. The various variations in rotation angles are solved using manual analysis.

No	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	End Effector Position (cm)
1	30°	30°	30°	$[13{,}66 \quad 13{,}66 \quad 0]$
2	45°	45°	0°	$[7{,}07 \quad 17{,}07 \quad 0]$
3	0°	90°	0°	$[10 \quad 10 \quad 0]$
4	120°	100°	30°	$[-12{,}66 \quad 2{,}23 \quad 0]$
5	−90°	−120°	30°	$[-8{,}66 \quad -5 \quad 0]$

Table 4. The Visualization and Analysis results with various variations in rotation angles (as Tabel 3) by MATLAB

No	Visualisation Result	Discussion	
		Angle Variation	End Effector Result
1		$\theta_2 = 30°$ $\theta_3 = 30°$ $\theta_4 = 30°$	$\begin{bmatrix} 13{,}660 \\ 13{,}660 \\ 0 \end{bmatrix}$
2		$\theta_2 = 45°$ $\theta_3 = 45°$ $\theta_4 = 0°$	$\begin{bmatrix} 7{,}071 \\ 17{,}071 \\ 0 \end{bmatrix}$
3		$\theta_2 = 0°$ $\theta_3 = 90°$ $\theta_4 = 0°$	$\begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$

4		$\theta_2 = 120°$ $\theta_3 = 100°$ $\theta_4 = 30°$	$\begin{bmatrix} -12,66 \\ 2,232 \\ 0 \end{bmatrix}$
5		$\theta_2 = -90°$ $\theta_3 = -120°$ $\theta_4 = 30°$	$\begin{bmatrix} -8,660 \\ -5 \\ 0 \end{bmatrix}$

Based on Tabel 3 and Tabel 4, it shows that the overall rotational motion influences the end effector of the transforming robot motion. For the cases, when the sum of angle for each link (θ_2+θ_3+ θ_4) is same but different for each θ_2, θ_3, and θ_4, we found that the end effector of the robot is different for each case.

CONCLUSION

Based on the results of the analysis and discussion, we can conclude that regarding the equation of the forward kinematics equation for the motion of the robot arm (manipulator) with 4-DoF obtained are as equation (10). The visualization of the system can be shown in Figur 3 and Tabel 4. Thus, the overall rotational motion influences the position of the end effector of the robot. For the cases with the same number of angles but with different θ_2, θ_3, and θ_4, it turns out that the end effector of the robot is different for each case.

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